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Note on a previous paper on Fermat's
last theorem

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NOTE ON A PREVIOUS PAPER ON FERMAT'S LAST THEOREM

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6. In a previous paper ¹⁾ a lower bound for z satisfying

$$x^p + y^p = z^p \quad (x, y, z \text{ positive integers; } p \nmid xyz; x < y < z)$$

was given. It here may be remarked that from the derived results easily lower bounds also for x and y can be found.

From the formulae

$$2x = a^p - b^p + c^p, \quad 2y = -a^p + b^p + c^p \quad (2.6)$$

one gets, using $0 < a < b < c$ and

$$c > \frac{p^3}{3 + \log 2p} - p, \quad (4.2)$$

the result $2y > c^p$, hence

$$y > \frac{1}{2}c^p, \quad c > \frac{p^3}{3 + \log 2p} - p. \quad (6.1)$$

Further one deduces from (2.6)

$$2x = a^p - b^p + c^p \geq a^p + p(c - b)b^{p-1} \geq a^p + pb^{p-1} > pb^{p-1}. \quad (6.2)$$

Using $b = \beta c$ and the formulae (4.2) and

$$\beta > 1 - \frac{\log 2pe}{p}, \quad (3.4)$$

one gets

¹⁾ A remark on Fermat's last theorem, H. J. A. Duparc and A. van Wijngaarden, Nieuw Archief voor Wiskunde (3) I, 123—128 (1953).

$$\begin{aligned}
b &> \left(1 - \frac{\log 2pe}{p}\right) \left(\frac{p^3}{3 + \log 2p} - p\right) = \\
&= \left(1 - \frac{\log 2pe}{p}\right) \left(\frac{1}{\log 2pe^3} - \frac{1}{p^2}\right) p^3. \quad (6.3)
\end{aligned}$$

Now

$$\begin{aligned}
\left(1 - \frac{\log 2pe}{p}\right)^{p-1} &= e^{(p-1) \log \left(1 - \frac{\log 2pe}{p}\right)} \\
&> e^{-(p-1) \left(\frac{\log 2pe}{p} + \frac{\log^2 2pe}{p^2} + \dots\right)} \\
&= e^{-(p-1) \frac{\log 2pe}{p} \frac{p}{p - \log 2pe}} = \left(\frac{1}{2pe}\right)^{\frac{p-1}{p - \log 2pe}}
\end{aligned}$$

and

$$\begin{aligned}
\left(\frac{1}{\log 2pe^3} - \frac{1}{p^2}\right)^{p-1} &> \left(\frac{1}{\log 2pe^3}\right)^{p-1} \left(1 - \frac{\log 2pe^3}{p^2}\right)^p \\
&> \left(\frac{1}{\log 2pe^3}\right)^{p-1} \left(1 - \frac{\log 2pe^3}{p}\right).
\end{aligned}$$

Consequently from (6.2) and (6.3) one obtains

$$x > \frac{1}{2} \left(\frac{1}{2pe}\right)^{\frac{p-1}{p - \log 2pe}} \left(\frac{1}{\log 2pe^3}\right)^{p-1} \left(1 - \frac{\log 2pe^3}{p}\right) p^{3p-2}. \quad (6.4)$$

From the bound $p \geq 253747889$ one then gets the same numerical lower bound for x , y and z , viz.

$$z > y > x > 10^{6 \times 10^9}. \quad (6.5)$$

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